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A letter on the present state of affairs

Prior, indeterminism and relativity 40 years later

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Abstract The paper re-evaluates Prior's tenets about indeterminism and relativity from the point of view of the current state of the debate. We first discuss Prior's claims about indeterministic tense logic and about relativity separately and confront them with new technical developments. Then we combine the two topics in a discussion of indeterministic approaches to space–time logics. Finally we show why Prior would not have to "dig his heels in" when it comes to relativity: We point out a way of combining the existential import of the distinction between past, present, and future with a frame-relative notion of the present.

Keywords Philosophy of time · A-theory · Relativity theory · Arthur Prior

Dear Professor Prior,

Since 1969, many things have happened. We are not quite sure how this letter is going to reach you (aerograms have come out of fashion, but paper-independent communication has made some progress). Still, we would like to seize the opportunity of a special edition of *Synthese* for telling you a bit about what has happened in a field we know you were especially interested in: the tense logic of relativity. We would like to link this with the topic of indeterminism: another topic you advanced so much. Connections between both topics can be seen much more clearly today.[1] Both of us

[1] This is certainly mostly due to Belnap (1992).

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did some work on them. We don't necessarily agree on every detail, but there is plenty of common ground to agree on what the overall developments and results are.

To mention the trickiest point first (as we know you will appreciate): Today, there is a tendency to accept some relativization of tense-operators to reference frames that is contrary to what you were willing to accept. You may not be entirely happy with that at first sight. But don't worry. We do share your concern about the existential import of the present that you stressed so aptly using the example of relief. Our everyday intuitions about that ("thank goodness that's over", Prior 1959) should definitely be saved. It's just that we do not think they are threatened by the kind of relativization just mentioned. So there is no need for "digging your heels in"[2] with respect to Relativity. We would like to convince you of that. We regard this as a continuation of your ideas you could be happy with, but which, on the other hand, will not need to face resistance among open-minded physicists. In short, we think we can suggest something that both you and a staunch but open-minded defender of Relativity orthodoxy like, e.g., Howard Stein could buy.[3]

Apart from that tricky point, let us state first that there are lots of news, from general to technical, that we are sure you will be simply pleased to learn. As to the general points:

- Metaphysics is a big issue again.[4] Analytic philosophy does not shun away from it any more. On the contrary: Both live together in harmony. May we call you one of the matchmakers?
- Indeterminism is considered a perfectly respectable position. It is gaining more and more influence.[5]
- Tense logic, always remembered as your invention, has become a vast area of research within modal logic and proved productive outside philosophy as well, e.g., in computer science and artificial intelligence.[6]

In this letter, we are not concerned with these general issues. But there are important news in the more limited fields on which we will comment below. Importantly:

- There are still A-theorists and B-theorists. So the A-series, which you defended not just against McTaggart's purported proof,[7] but also in other ways, has by no means been done away with. More than that, we believe that the A-series is in excellent condition, alive and kicking. Your views about its importance and irreducibility have turned out right, along with many of your ideas about "I". Those papers in which you favour the irreducibility of "now" got it right.[8]

[2] As you put it in your essay "Some Free Thinking About Time" (Prior 1996).

[3] Cf. Stein (1991).

[4] By the way, even social ontology is. However, your 1937 paper on "The nation and the individual" (Prior 1937) that defines the field is still not as widely known as it deserves.

[5] To name but a few contemporary writers who have contributed to developing that position: Belnap (1992), McCall (1994), Placek (2000), Belnap et al. (2001), Perry (2006).

[6] Cf., e.g., Øhrstrøm and Hasle (1995) as well as a numbers of papers in Copeland (1996).

[7] Cf., e.g., Prior (1967, Chap. 1).

[8] Despite some remarks in your last paper, "The notion of the present" (Prior 1970), we take it that the redundancy theory of the present is a position you would probably not have upheld.

The importance and irreducibility of perspectival world access is widely acknowledged today.[9]

– Peircean indeterministic tense logic has been completely axiomatized,[10] Occamistic tense logic most probably has.[11]
– The modal-logical system S4.2 is not just correct, but even complete for special relativity with the reflexive signal relation "lies in the forward light-cone of or is identical with" as the accessibility relation.[12] Recently, a complete axiomatization for the corresponding irreflexive accessibility relation has been found as well.[13] While it may not be entirely clear which of them you meant in your sketch in *Past, Present and Future*,[14] both have worked out fine: congratulations!

We would now like to present specific developments in three sections:

(1) Indeterministic tense logic
(2) Tense logic for relativity without modalities
(3) Tense logic for relativity with modalities.

Once we have informed you about these, we will be able to position ourselves with regard to your remarks on absolute time-order in "Some Free Thinking about Time" (cf. note 2; we hope you don't mind this piece has been published). After that (4), we will be able to make our own suggestion as to how to reconcile your views with Relativity by distinguishing three different kinds of relief.

1 Indeterministic tense-logic

Occamistic and Peircean indeterministic tense-logic (both of them your invention) have received a lot of attention both technically and philosophically, and both have found adherents. Today, both types of logics are mostly discussed within the framework of combined temporal and modal logics, which takes both instants of time and, separately, possible world histories as primitive. That might seem to you to be ontologically over-generous, since you recommended to not even take instants of time ontologically seriously, let alone world histories. Technically, it does not make much of a difference: the primitives you call "instants", are now often (but not always) represented as pairs consisting of some instant of time and some alternative. You will easily get used to that, as well as to infix notation instead of Polish notation which, alas, proved to be unteachable in the long run, despite your good advice on how to read it.[15] We are aware of your skepticism towards model theory. Despite the fact that

[9] We still think many more people should read your great paper on perspectival world access, "Identifiable individuals" (Prior 1968b, pp. 66–77).

[10] Burgess (1980).

[11] Reynolds (2003).

[12] Goldblatt (1980).

[13] Shapirovsky and Shehtman (2003).

[14] Prior (1967, pp. 203–205).

[15] We're alluding to the preface of your *Past, Present and Future* (Prior 1967, p. v).— In fact, the second edition of your *Papers on Time and Tense* (Prior 2003), which was put together following your drafted list

most of the discussion today is in terms of models, you clearly had something there: Your Quine-critical discussion of four "grades of tense-logical involvement" (Prior 1968b, p. 116) has inspired many.[16] And following your work and the work of Kit Fine, who did the logic community such a good service in publishing *Worlds, Times and Selves* (Prior and Fine 1977), the question of intertranslatability of model-theoretic and object-language level semantics resurfaced in the development of what is now called "hybrid logic".[17] Intertranslatability of technical results thus presupposed, we consider the philosophical debate still open. Your remark that models could just have the status of "handy diagrams" (Prior 1967, p. 75) seems to us to point in the right direction.

Your basic insight in developing tense logic was that one could evaluate formulae at instants connected by a temporal ordering, in a way formally similar to the semantics for modal logic that were being developed at that time.[18] Instead of one pair of a strong ("necessarily") and a weak ("possibly") modal operator, in tense logic you introduced two such pairs of operators, the weak ones being P (past, for "it was the case that") and F (future, for "it will be the case that"). The basic problem of tense logic, one might say, is how to understand the F operator in a way that does not deny your (and our) indeterministic intuitions, i.e., how to define truth-values for future tensed formulae based on time structures that are not just linear.

Your key insight in defining the Peircean and Occamistic systems was that in branching time structures (remember Kripke's letter from 1958?), the concept of a history (a maximal linear path *h* through the structure) plays a really crucial role. As you will remember, in the Peircean system, $\mathbf{F}\alpha$ ("it will be the case that α"; we use boldface here to indicate the Peircean operators) is always assigned an *actual* truth value: that formula is considered true at an instant *t* if and only if α will be true at some later point in *all* histories that are still open at the instant of evaluation. Thus, the future operator "**F**" expresses something like settled future truth. Peirceanism allows for a clear and intuitive distinction between "$\neg\mathbf{F}\alpha$" ("it is not settled that α will be true sometime") and "$\mathbf{F}\neg\alpha$" ("it is settled that α will be false sometime"), based on which one can also define an operator "**f**", "it may forever be the case that", as "$\neg\mathbf{F}\neg$".

Footnote 15 continued
for such an edition, has infix formulae in place of your Polish notation, and it seems to be doing your ideas a good service through this.

16 Cf., e.g., Øhrstrøm (1997). By the way, the days when Quine's ideology ruled the philosophy of logic are now over—and in part that is certainly due to your kind and stubborn opposition. At one point you say that Quine got (just) two things exactly right in that area, one being an idea about rendering propositional attitudes (Prior 1971, p. 20). May we ask what is the second point? One of us has ventured the guess that it's his motto for *Word and Object*, but we'd be curious to learn what it really was.

17 Cf., e.g., Blackburn (2000). The area is developing fast, with many interesting applications in computer science and elsewhere, and thus, you have posthumously become the founding father of yet another logical discipline!

18 Writers on the history of modal logic (Copeland 2002; Goldblatt 2005) now cite your early work together with Carew Meredith (published in Copeland 1996, p. 133f.) as one of the founding moments in this development. Despite your qualms about the model-theoretic tradition in which much of that development has taken place, we assume this would please you.

On the whole, Peirceanism seems to have been the position you favoured. It is certainly still attractive. However, it seems to face difficulties in accounting for a number of important and clear ways of using the future tense: When something contingent has happened, one can say *afterwards* that "this was what was going to happen" — but if "$\mathsf{F}\alpha$" is false before the event, "$\mathbf{PF}\alpha$" can still be false afterwards.[19]

You surely had reasons to be sceptical of the Occamistic position, at least as far as it was developed in the late 1960ies. The crucial feature of the Occamistic approach to tense logic is that $\mathsf{F}\alpha$ ("it will be the case that α") is evaluated with respect not just to an instant t, but also with respect to some history h passing through that instant. (One standardly also defines an operator for historical necessity, "N", that quantifies universally over all available histories; the Peircean "$\mathbf{F}$" is then definable as "NF".) As one might expect, $\mathsf{F}\alpha$ is considered true iff α is true later on at some instant in *that* (linear) history h. Which history? That turned out to be the key question in the further development of Occamistic tense logic. Naively assuming the history to be "the one that will in fact happen" appears to beg the question against indeterminism—but is there another way?

Speaking in terms of "parameters of truth",[20] Occamism takes both a time of evaluation t and a history of evaluation h to be a parameter of truth. The problem for an indeterminism-friendly Occamism is that a context of utterance, while it *does* specify a time of the context t_c to initialize the t parameter, does *not* specify a value h_c for the h parameter.[21] How can one deal with the uninitialized history parameter h? If the h-dependence of future tense statements simply means that such statements are "true *just in case* history h is realized", statements about the future are always completey iffy. This is at odds with our usage of the future tense, according to which we seem to assign truth-values to future tense statements, or at least try to. Model-theoretic metalanguage ("$\mathsf{F}\alpha$ is true at t in h") does not provide a way out here either—as you stressed many times, we speak *our object language*. Thus, for there to be truth values, there has to be a way of initializing the h parameter. Initialization by "assuming h to be the *true* future history", perhaps in the form of a "thin red line" (Belnap et al. 2001, Chap. 6), may be a nice metaphor, but it seems to amount to either determinism in sheep's clothing or loose talk. (Not everybody agrees with this assessment—but let's look for alternatives in any case.) Fortunately, a number of variants of Occamistic indeterminstic tense logic have been formally specified since 1969; let us tell you about three specific ones. One of them is an application of Free Logic to indeterministic tense logic, using the technique of supervaluations (Thomason 1970)—incidentally, people use the term "Prior-Thomason indeterministic tense logic" variously for any generally Occamistic approach or for its supervaluationist variant. Another line of making sense of Occamism is an attempt

[19] We are here cutting short some of the intricacies of your discussion of the Peircean system prompted by your reliance on metric tense operators (Prior 1967, Chap. VII). While these have proved very useful in technical applications, the philosophical discussion has—for better or worse—mostly shunned away from them, and thus we have only considered non-metrical Peirceanism here.

[20] Cf. Belnap (2005b, 2007) for two lucid accounts of how to handle parameters of truth in a uniform way.

[21] Belnap et al. (2001, Chap. 6E), draws an interesting technical parallel between assignments of values for free variables and the history parameter: *both* are "mobile parameters that are not initialized by context" (p. 173). Maybe you would have liked the accompanying suggestion, which goes against Quinean extensionalism, that quantification is, after all, not that extensional a notion.

to make systematic use of the time the content of an utterance pertains to, and to differentiate the semantics of the "F" operator accordingly. A third approach stresses the fact that assertions about the future, as well as bets, promises and other future-related activities, are human actions. According to this approach, searching for truth-values for all future-related utterances is rejected as misguided, but a systematic theory of backward-looking truth-value attributions is provided. All of these approaches employ a distinction between a context of utterance (the speaker's perspective) and a context of evaluation—a point that is familiar to you, since you discovered it (along with Hans Kamp) in your formal investigations of "now" (Prior 1968a).

The supervaluationalist approach evaluates formulae at (t, h)-pairs like the naive Occamistic indeterministic tense logic sketched above. However, it does not take the values this procedure yields seriously as truth-values, but only the so-called supervalues based on them. So α is true at t (irrespective of any initialization for h) iff its Occamist value is 1 in *all* alternatives h that are still open at t, and false, iff the Occamist value is 0 in *all* of them. If the Occamist values are mixed, α's truth-value is undefined. This approach leaves all laws of classical propositional logic intact, but assesses "$\mathsf{F}\alpha \vee \neg\mathsf{F}\alpha$" as being true even when neither disjunct is, i.e., in the case of future contingents. It also captures the perspectival aspects of future contingents: *Before* the event, the truth value of "$\mathsf{F}\alpha$" is undefined, but after "α" has been made true, a backwards-looking "$\mathsf{PF}\alpha$" ("it was the case that α was going to obtain") is evaluated as true because *then* only "α"-alternatives are taken into consideration.[22]

The second approach was called "hemiactualism" by its inventor Kimio Harada.[23] The idea is to assign to every formula a definite truth value, but both relative to a position of utterance and a position of evaluation. If the position of utterance is before the event, the F-operator is taken to behave like a Peircean operator; if afterwards, it behaves like an Occamistic operator. Thus, "$\mathsf{PF}\alpha$" is true after a truth-maker for α has occurred, even though "$\mathsf{F}\alpha$" is false before (and "$\neg\mathsf{F}\alpha$" is true). No truth-value gaps occur. Axiomatization for this approach has not been researched into.

In a somewhat similar vein, Nuel Belnap also stresses an idea of "double time references":[24] In a branching structure, a later moment singles out a bundle of histories that can be used to assess bundle-relative settled truth with respect to an earlier moment of evaluation. However, Belnap sticks to a uniform semantic account rather than Harada's Peircean switch, and he stresses the pragmatics of the normative embedding of our uses of future tense statements. *This* is, or so he argues, what takes care of the openness of the history parameter: "assertion involves a closing of the history parameter in the sense that assertion is an act that has implications for the speaker and others no matter how things eventuate" (Belnap et al. 2001, p. 174f.). Thus future tense sentences (typically) have no truth value, but this feature is perspectival as to the position of utterance. Like in the supervaluationist picture, *afterwards*, "$\mathsf{PF}\alpha$" is true,

[22] As one would hope, "$\mathsf{PNF}\alpha$" ("it was the case that necessarily, α was going to obtain") still turns out false for contingent α. It is still a subject of debate whether supervaluationism is fully adequate with respect to the perspectivity of our tensed language. Cf., e.g., MacFarlane (2007), who gives an ultimately negative verdict. Along similar lines, cf. Belnap et al. (2001, p. 156), and MacFarlane (2003).

[23] Harada (1994).

[24] Cf. Belnap et al. (2001, Chap. 6) and Belnap (2001).

because *then* only "α"-alternatives are still open and relevant in the sense of double time references.[25] And furthermore, the future-tense sentences *are* assertable, because they always have clear pragmatic upshot. In view of your interests in action theory, maybe you would have liked that approach?

2 Tense logic for relativity without modalities

The way Special Relativity is commonly interpreted, the B-series notions "earlier", "simultaneous", and "later" are never to be understood absolutely, but only relative to a given frame of reference. Of two distant (i.e., space-like separated) events that are simultaneous relative to one frame of reference, one may be earlier than the other relative to another frame, and the order of such events may even be reversed from frame to frame.

Relativizing the B-series notions might well affect the A-series notions "past", "present" and "future". For do not just those events that are simultaneous with an event e, form the "slice" of those events that are present once e is present? And don't all earlier slices form the past, all later ones the future then? So if the B-series notions are relativized, will not the same have to happen to the A-series notions?

There are several ways in which one can react to this challenge.

1. One may say that the relativization of the B-series notions is inescapable, while the relativization of the A-series notions is intolerable, but that the latter follows from the former. In that case, the situation should be interpreted as a *reductio ad absurdum* of the A-series: The only way to escape absurdity is denying the reality of the A-series.[26] It is clear that you don't share this view. Neither do we.
2. One may say that the relativization of the B-series notions is inescapable, while the relativization of the A-series notions is intolerable, but that the latter does not follow from the former. In that case, relativized B-notions might co-exist with absolute A-notions.[27] This position faces the challenge that there might be no way of distinguishing the *real* present from any other space-like hypersurface containing a given event.[28]
3. One may say that the relativization of the B-series notions is intolerable, because the relativization of the A-series notions is intolerable and their absoluteness therefore inescapable, while the latter follows from the former. In that case, the situation should be interpreted as a *reductio ad absurdum* of the relativization of the B-series: Neither *real* simultaneity nor the *real* present are relative, but both are absolute. Again, the trouble is that this position might be epistemically empty

[25] Additionally, unlike the supervaluationist, on this proposal one is justified in saying that the former *utterance* of "Fα" was *true*. However, the philosophical import of that fact is debatable; cf. the references in note 22.

[26] Cf., e.g., Mellor (1998).

[27] Cf., e.g., Rakić (1997a, b).

[28] In Special Relativity, such a hypersurface will have the form of a three-dimensional Euclidean space. In General Relativity, such a hypersurface could take the wildest shapes (if only we could show you the wonderful volcano landscape Pete Forrest drew on the blackboard in Kirchberg in August 2005!).

because there might be no way of singling out the *real* simultaneity hypersurface of a given event.

4. One may say that the relativization of the B-series notions is inescapable, which makes the relativization of the A-series inescapable, too, since the latter follows from the former. Contrary to position 1, on this position one would accept a relativized A-series (as well as B-series, of course). It is clear that you do not share this view. Indeed, the relativization of the A-series is a hard bullet to bite. One should have some detailed story to tell why this does not destroy our everyday intuitions about the existential import of the A-series, e.g., with respect to our experiences of relief.

Given your view that the B-series is derived from the more fundamental A-series,[29] it seems that you strongly favoured position 3. However, some of your views may be compatible with both positions 2 and 3:

> When an event *X* is happening, another event *Y* either *has* happened or *has not* happened — "having happened" is not the kind of property that can attach from one point of view but not from another. (Prior 1996, p. 50)

> [W]e ask whether this next pulsation, although we won't of course observe it for a while, is in fact going on right now, or is really still to come, or has occurred already. On the view of presentness which I have been suggesting, this is *always* a sensible question. […] The difference between pulsating — really and actually pulsating — and merely having pulsated or being about to pulsate, is as clear and comprehensible a difference as any we can think of, being but one facet of the great gulf that separates the real from the unreal, what is from what is not. Just this, however, is what the special theory of relativity appears to deny. (Prior 1970, p. 247f.)

> So it seems to me that there's a strong case for just digging our heels in here and saying that, relativity or no relativity, if I saw a certain flash before you, and you say you saw it first, one of us is just wrong […] even if there's no physical means whatever of deciding which of us is. […] [T]he theory of relativity isn't about real space and time, in which the earlier-later relation is defined in terms of pastness, presentness, and futurity […] (Prior 1996, p. 50f.)

What do we think about this?

In fact, a pretty good case can be made for positions 2 and 3. Along the lines of a suggestion made by Øhrstrøm (1988), one can argue that what *looks* like their common trouble might in fact not *be* any trouble, because we might be in a better position for distinguishing an absolute, real present than one might think with only *Special* Relativity (SR) in mind: Cosmologists work with models for General Relativity (GR) that allow for the definition of an absolute cosmic time.[30] You have sketched a "logic of points of view" that might be adequate for this position, and one of us has tried

[29] Cf., e.g., the quote from Prior (1996) below, as well as the discussion about "handy diagrams" referred to above.

[30] Some further references can be found in Müller (2002, pp. 241–247, 250f.).

to develop these ideas in some formal detail.[31] In that "absolutistic logic of standpoints", operators "A_S" are read as "it appears from the point of view S that", and propositions are place-indexed. It may well happen that, e.g., "$A_{S_1}\mathsf{F}\phi$(Sirius)" and "$\neg A_{S_2}\mathsf{F}\phi$(Sirius)" are both true. The "true" point of view S_0, embodying the absolute present, is accordingly expressed by an operator A_{S_0}, and $A_{S_0}\mathsf{F}\alpha$ is true iff $\mathsf{F}\alpha$ is true *simpliciter*.

While this marks a coherent position, one might also be unconvinced that SR should be played against GR in this way: There is clear evidence that GR beats SR in terms of empirical adequacy (our current satellites, such a help for long-distance communication when compared to the 1960s air mail letters, are adjusted to effects of GR), but the debate about an adequate cosmological model and thus, about the viability of a cosmic time is still open. If one wishes to stick to SR when it comes to simultaneity—against what you held dear—option 4 above certainly looks more promising.

And at any rate, as you also pointed out in your sketch of a "logic of points of view", it is important to capture a way of speaking in which A-series notions *are* relativised to a perspective. You were ultimately critical to that attempt and attested it "a solipsist ring" (Prior 1968b, p. 134). However, we hope to be able to convince you otherwise—we think that in this respect, you followed the physicists' talk of "causal time" too closely (see below).

The project of a "relativized logic of points of view" can be approached in various ways. We have both tried our hands at it. Müller has opted for following your sketch of such a system quite literally, which means that his system works with something like metric tense operators. Since a point of view is, in SR, not just a point in time, but both a space–time point (defining a "here-now"), a spatial orientation and a frame of reference, the "metric" operators take ten real parameters now instead of the one integer parameter of your metric tense operators: Any element of the proper orthochronic Poincaré group corresponds to a possible change of the point of view. This may be complicated, but it appears to be the only way to save your claim that "given how the course of events appears from a certain point of view, [one] will be able to calculate how it will appear from certain points of view" (Prior 1968b, p. 133). Note that in this approach, tense and place operators are just special cases of operators affecting the current point of view. Shifts from one reference frame to another via Lorentz transformations are thus interpreted as general changes of *perspective*. It's *your* emphasis of perspectival world access along with *your* skepticism about model theory and *your* high esteem of the A-series that motivated this suggestion. If the relativization of A-series notions is so well manageable, that seems rather a point for than against an A-theory of time.

Strobach uses quantifier-like pairs of modal operators separately for changes of the frame of reference and for changes of place: "for every reference frame" ("$\times$")/"for some reference frame ("$+$") and "everywhere" ("E")/"somewhere" ("S").[32] Reference frames determine a foliation of space–time into space-like simultaneity hypersurfaces

[31] Your sketch, as you will remember, is just one aspect of your immensely rich paper on "Tense logic and the logic of earlier and later" (Prior 1968b, p. 133f.). A reconstruction is attempted in Müller (2002, Chap. 4.5.3), and, somewhat differently, in Müller (2004).

[32] Cf. Strobach (2007a, Part III). For a summary, cf. Strobach (2007b).

and also determine which events occur at the same place. The light-cone structure is invariant under changes of the frame of reference. Thus, the only event that *e*'s past and future light-cones have in common with any of its (frame-relative) simultaneity hypersurfaces is *e* itself, while all events locotaneous with *e* lie wholly inside *e*'s past and future light-cones.

Why all the trouble? Why not just take up your suggestion of a tense logic for Relativity with *causal* operators ("F_c" = "somewhere in the future light-cone" = "somewhere in the causal future"/"P_c" = "somewhere in the past light cone" = "somewhere in the causal past")? In a way our approach does account for this. But in a way we're also sceptical, since the causal tense operators cannot be fundamental, for metaphysical reasons.

Causal "tenses" can be defined in the system of the logic of points of view. Especially given the recent formal completeness results,[33] the causal operators provide an important formal constraint for any relativistic tense logic: Whatever operators one defines as basic, one should be able to define causal operators using these basic operators—and for scenarios for Special Relativity, the reflexive ("Diodorean") variants of the causal operators so defined must turn out to be S4.2-operators. This turns out as it should: in terms of the above formalism, "F_c" may indeed be defined as "$+\mathsf{F}+$" (i.e., the causal future corresponds to frame-relative future sandwiched between two changes of frame).

On the other hand, one shouldn't be restricted to just the causal operators. They reflect what many physicists say but what we don't believe (and you expressed your worries about this too): that from Special Relativity itself it follows that the present shrinks to one single event. According to this view, that theory neither allows for any kind of spatially extended present nor for any meaningful talk of past and future outside the light-cones. We do not think that this follows from SR, but is just a wrong philosophical interpretation of it.

There are at least three theoretical options that accord with SR but won't shrink the present:

1. Assume an absolute spatially extended present as an element of reality that is just not distinguishable by means of SR (even though possibly distinguishable by other physical means).
2. Assume an absolute spatially extended present as an element of reality that is not distinguishable by *any* physical means.
3. Rather relativize and multiply the present than shrink it. On this view, there is a spatially extended present—one per reference frame.

Our guess is that, in spite of your (often well-founded) skepticism towards physicists' metaphysics, you concentrated on causal operators because that was what was being discussed at the time.[34] And the causal relation provides as much absolute temporal order as one can get from Relativity: within the light-cone, all reference frames agree on the order of succession of events. To both of us, the price of shrinking the present

[33] Goldblatt (1980), Shapirovsky and Shehtman (2003).

[34] Your reliance on the "signal relation" from Carnap's *Introduction to Symbolic Logic* at least points strongly in this direction (Prior 1968b, p. 133f.).

seems too high, so that we'd urge you to consider one of the options 1–3 above (we don't fully agree on what would be the best pick ourselves, so there is certainly some leeway for discussion).

Maybe at this point it is instructive to develop an interesting formal parallel between relativistic tense logic and indeterministic tense logic for branching time: There are, in fact, direct formal analogues of a naive Occamist, a Peircean, and a supervalutionist position in relativistic tense logic. This comes about naturally if one takes a frame of reference f as an additional parameter of truth of a formula, as discussed above. However, this parameter is rather like the (unproblematic) t parameter than like the (difficult) h parameter, since whichever initialization for f one chooses, one will not make a factual error, and often the current rest frame of the person who utters the formula is a perfectly natural choice. This is independent of the question of whether there is a preferred, "true" frame of reference f_0.[35] So what should be the relativistic analogue of "it was the case", "it is the case" and "it will be the case"? Since we are talking about relativity without modalities here (we'll discuss modalities in the next section), the future operator does not pose any specific problems. All operators will be suffixed by the "somewhere" operator, "S", to avoid any problems of frame-relative places—we want to focus on the purely *temporal* aspect here. There appear to be at least five options:

- One can use frame-relative tense-operators "PS", "S", and "FS" without presupposing a preferred reference frame. Semantically, this roughly corresponds to Occamism in branching time (even though, as discussed above, the frame parameter is unproblematic).
- One can use frame-relative tense-operators "PS", "S", and "FS", but base these semantically on a preferred reference frame, f_0. This is an analogue of the naive Occamist, "actualist" (i.e., deterministic) position of presupposing a "thin red line".
- One can do away with frame dependence by quantifying over frames universally. In terms of the operators discussed above, this means employing "$\times$PS", "$\times$S", and "$\times$FS". Here we are clearly speaking about an analogue of the Peircean position, and even the point about Peirceanism being definable in terms of the Occamist operators plus historical necessity has a direct formal analogue: Just as the Peircean "**F**" for branching time could be defined as "NF", so we here have a Peircean combination of a strong operator "$\times$" and the parameter-relative Occamist operators, "PS" etc.
- A weak Peircean reading that quantifies over frames existentially is also possible: For this, one would use the operators "+PS", "+S", and "+FS".
- Finally, supervaluation over all reference frames corresponds to supervaluation over all future alternatives. In the relativistic case, this may lead to truth-value gaps with regard to happenings outside the light-cones.

[35] Remember that with respect to the history parameter h, assuming that there *is* a true future history h_0 arguably amounts to a denial of indeterminism. Assuming the existence of some f_0, on the other hand, has no such dire consequences—it may just be epistemologically suspect, as discussed above.

In line with our observation about the different status of the history and the frame parameter, the technical analogue to an intuitively attractive position in indeterministic tense logic need not be an intuitively attractive position in relativistic tense logic (nor the other way around): Peirceanism in relativistic tense logic is less attractive than in branching time, since it amounts to shrinking the present to a single event. On the other hand, the Occamistic variants are much more straightforward in relativistic tense logic than in branching time, since they unproblematically reflect the idea of a multiplicity of reference frames. And actualism might be less absurd in relativistic tense logic than determinism is in branching time.

But, as you will probably say, *all* of this except actualism is intuitively entirely unattractive! The present demarcates the realized from the unrealized, the one definite past from a future of many alternatives. And how should one relativize that? Think of branching. " 'Having happened' is not the kind of property that can attach from one point of view but not from another"—that isn't just clear, intuitive "free thinking about time", as you put it, but also seems like a strong argument. Let's look more closely at how indeterminism and Relativity can meet. For only then will branching be able to play the role it deserves.

3 Tense logic for relativity with modalities

In 1992, Nuel Belnap published a paper that has since turned out to be epoch-making. You will remember Belnap as a fair sparring partner over the knotted issue of your "tonk" connective,[36] as well as from the personal discussions that you had at Alan Ross Anderson's place, at the time before you stopped traveling to the U.S. in protest against the Vietnam war. Belnap's paper, "Branching space-time" (Belnap 1992, henceforth BST), provides models for indeterministic space–time structures. That rich paper links suggestions from the interpretation of quantum theory with Relativity and even makes some first suggestions for treating the much-discussed case of quantum-mechanical Einstein–Podolsky–Rosen correlations. BST has triggered a lot of research, both formally and with a view towards applications.[37] The paper could certainly not have been written, had you not established research into branching *time* in the first place.

BST models resemble your models (if we may say so) for branching time (BT) in that they take possible point events as primitive entities where BT uses "instants" in the sense of tempo-modal positions. These possible point events are not connected via some absolute earlier/later-relation with no regard for spatial position, but by the (likewise) absolute signal relation "lies in a possible future light-cone of". Just like in BT models with their semilinear partial ordering relation, the BST partial ordering is not linear: Possible point events may lie in the same alternative, but be space-like separated from each other. Or they may occur in different alternatives. In the first case they will have some other possible point event as their common upper bound. This

[36] Cf. Belnap (1962).

[37] Cf., e.g., Belnap and Szabo (1996), Müller and Placek (2001), Belnap (2003, 2005a), Müller (2005, 2006), Weiner and Belnap (2006), Müller et al. (2008).

restricts BST models to scenarios that are in some sense close to SR, as you showed in advance by pointing to what is today often called trousers worlds:[38] scenarios of GR where light cones get bent away from each other due to gravitation, so they don't have a common upper bound even within the same alternative. This restriction is a price one has to pay for the elegance of doing with one domain of primitive entities and one ordering relation.

BST models allow for many different applications and interpretations. Fundamental branching is taken to occur at localized point events, which may however be coordinated (modally correlated) in various ways. Thus, BST models are compatible with assuming an absolute present which must have the same shape in all alternatives to enable ordered branching;[39] but using them is just as well compatible with thinking of "straight" frame-relative branching.[40]

So if you stick to your position you can use BST models and regard them as extensions of your own BT models, with their branching just as ontologically robust and A-theoretical as that of BT models.

One of us, however, makes a different suggestion: It is metaphysically attractive to assume that the fundamental branching takes place exactly along the edges of past light cones.[41] (By the way: "branching" is to be taken *very* metaphorically here.) The frame-relative branching of alternatives along some frame-relative present slice is *construed* out of overlapping branchings along the edges of the past light-cones the tips of which form the slice: We ourselves synthesize the suitable alternatives from *local* decisions whenever we tell a spatial story. Frame-relative branching is as home-made as the reference frames are. How is *this* going to persuade *you*? Please have a look at the raw material the spacewide alternatives are synthesized from. These are *local* decisions that are as real as any A-theorist could want them to be: You simply notice that *this* is where (how far) you've got on your world-line, you know that there is exactly *this* past light-cone that influenced that *this* is *how* where you've got has turned out. Something analogous applies for particles except that they don't do any noticing. At the local level, but only at the local level, past, present, and future are brute facts.

This is all a logical continuation of what you started: It was you who drew attention to the fact (already clear to Aristotle and medieval tense logicians) once more that there is not only logical necessity, but also a kind of positional necessity: the historical necessity of what is definite *now*. But this is only half the way: Relativity leads us to a different kind of positional necessity of what is definite *here and now*. It is impossible to represent deictic determinateness such that it can be read off the drawing for more than one event. "But", you will say, "I don't want to go all *that* way." There might be two reasons for saying that:

1. Doesn't that mean that what feels definite here-now might be indefinite from the position of some other, distant event in the space-like of my here-now?

[38] Cf. Prior (1967, p. 205).

[39] Cf. Rakić (1997a).

[40] Cf. McCall (1994).

[41] Whether BST models allow for this or not. For details of the suggestion, cf. Strobach (2007a, Part IV).

2. Doesn't that mean that what feels indefinite here-now is already definite from the position of some other, distant event in the space-like of my here-now?

As to 1: Yes, such things happen. But why worry? Whether it's definite or not here-now is something I will immediately notice here and now. This is A-theory, after all! If what is happening here and now is not definite somewhere else now, that is no more worrying than that it wasn't so here yesterday.

As to 2: That would be determinism, and thus very worrying. But the way space-wide present-slices and alternatives are synthesized this can never happen (my here-now cannot be included in the past light-cone of any event space-like related to it).

If you don't like it at first sight, try to look at it this way and you might like it at second sight: All the diagrams you can draw of past light-cone branching are *perspectival*. This *is* A-theory, after all.

4 Relief

Here is something a little less extreme in order to persuade you to dig your heels out again. Let us stress once more that the way you treat relief seems methodologically most recommendable: If there is some established fact of human existence that conflicts with physics, it is more reasonable to look for a more plausible interpretation of the bit of physics it clashes with than to deny the existential fact and what it presupposes. Sometimes, however, less reinterpretation of physics is needed for saving the existential facts than one might at first expect. And sometimes, not all everyday intuitions can be kept. Elevating just any of them to the status of an existential fact would only result in hampering physics. The earth's losing its status as the centre of the universe surely resulted in lots of existential shake-up. So what should be given up, and what should stay, in the clash between temporal intuitions and Relativity?

We believe that when it comes to relief, in fact less physics is affected than you would have thought: Relativity, taken as A-theoretically as we like to take it, can well accommodate the existential fact of relief, *even if we relativize A-notions to reference frames*. However, a little revision of everyday intuition is sensible and will, at a closer look, turn out to be existentially harmless. The idea is that three kinds of relief with respect to some happening—e.g., a decision about one's job application—should be distinguished:

1. the relief concerning the fact that one is not able to influence the decision any more ("Thank goodness I mailed this job application—nothing I can (nor need to) do now");
2. the relief concerning the fact that some decision has in fact been taken ("Thank goodness they've just made their decision; if they ever decide to hire me then they did so right now");
3. the relief concerning the fact that news of the actual decision has reached one ("Thank goodness I got the answer that they decided (not) to hire me").

If something analogous takes place where I am, i.e., on my world line, then all three kinds of relief coincide with the event of the decision (somewhat idealized, of course). What happens here may be influenced as long as it hasn't happened, it happens right

here, and it is known at once. This will also be the same for two observers who take exactly the same path through spacetime, but use two different reference frames for describing what is happening. But what if the decision takes place somewhere else?

Already in classical space–time, if signals travel at finite speed, the three kinds of relief can be distinguished. In fact, our above list illustrates this perfectly well (just suppose that regular mail is the only available means of communication). Once one takes Relativity into account, two things change: Firstly, the speed of signal transmission becomes limited *in principle*, and thus (1)–(3) differ in principle. Secondly and more importantly, two observers who choose different frames of reference can fail to agree with respect to (2). Both will feel the first kind of relief at the same occasion, i.e., the moment when the application is mailed. And both will feel the third kind of relief at the same occasion, i.e., the moment when they receive the answer. However, the first observer may consider himself justified to feel the second kind of relief earlier than the second one. In fact, you don't need *two* observers: Everybody can choose to use whatever reference frame he or she likes. So it is simply up to the observer when he feels like feeling the second kind of relief. Is this existentially worrying? No. It would be worrying if that happened with any of the other kinds of relief. But that is impossible.

Let us conclude by expressing our gratitude. Neither of us will want to miss the moment when he came across your work. It influenced our work and changed our lives. Dear Arthur (if we may), we owe you a lot!

All the best, yours,
Thomas Müller & Niko Strobach

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